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Aerodynamic noise dependent on mean shear

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New factors illustrating the effect of convection on acoustic radiation are deduced from Lighthill's theory of aerodynamic noise. For sound dependent on mean velocity variations for its generation, that is shear noise, the new factors are $(1-M\cos\theta)^{-3}$ and $(1-M\cos\theta)^{-1}$ in contrast with $(1-M\cos\theta)^{-5}$ for self noise.

Comparison with the measured far-field directionality patterns reported by Howes (1960) shows that the factor $(1-M\cos\theta)^{-3}$ provides considerably better agreement with the experimental observations than the self noise factor.

The source terms discussed by Lighthill (1954) when considering shear amplification are examined and shown to contain the terms produced by the analysis of Csanady (1966) as well as a new term dependent on the second derivative of the mean velocity.

1. Introduction

Lighthill (1952) from his representation of aerodynamic sources as a distribution of quadrupoles placed in a medium at rest deduced the U^8 law. If the sources move, a convection amplification effect occurs, increasing the velocity dependence over and above the U^8 law. The experimental results, on the other hand, show a velocity dependence close to U^8. In his second paper Lighthill (1954) showed that a fraction of the noise is proportional to the mean velocity gradient and this leads to the classification of noise which is generated only in the presence of a mean velocity gradient, as shear noise, and of noise which is generated by turbulent fluctuations themselves without the necessity of mean velocity variations, as self noise. Lighthill suggested that pressure fluctuation in the presence of mean shear was the principal mechanism of noise generation but Csanady, starting from a slightly different point in Lighthill's theory, showed that Reynolds stresses were potentially as important as the pressure fluctuations.

In the present paper it is shown that the Reynolds stresses are contained in a term set aside by Lighthill and how still another potential acoustic generator, pressure and Reynolds stresses in the presence of the second derivative of mean velocity, is implied by the quadrupole stress tensor. At the same time the convection amplification factor is found to be different for the self noise terms, the shear noise terms involving the first derivative of the mean velocity and the shear noise terms involving the second derivative of the mean velocity.

In the next section Lighthill's expression connecting the acoustic fluctuations to the stress tensor T_{ij} is used and the stress tensor approximated by the density–

velocity product, $\rho u_i u_j$. Lighthill's most convenient expression is that where the time differentiation of T_{ij} and the volume integration are performed in a frame of reference moving with the stress fluctuations. The volume integration is not evaluated instantaneously but at a retarded time resulting from the difference of the time of emission between the front and rear of an eddy. It is more convenient to evaluate Lighthill's expression in a moving frame rather than a fixed one because in the moving frame the influence of retarded time is more explicit. To a fixed-frame observer situated at an angle θ to the direction of motion of an observer moving with a convection velocity aM (where a is the speed of sound), the effective parts of both the time differentiation and the volume integral appear a factor $(1-M\cos\theta)^{-1}$ larger than they do to the moving-frame observer.

The volume integral contributes one factor $(1-M\cos\theta)^{-1}$ so that the self noise fluctuations, which contain a second time derivative, have a convection factor $(1-M\cos\theta)^{-3}$. The shear noise terms involving the first time derivative and a fixed-frame mean velocity gradient have a factor $(1-M\cos\theta)^{-2}$ while the terms involving the stress fluctuations and the second derivative of the mean velocity have the factor $(1-M\cos\theta)^{-1}$. Both the shear noise convection factors produce a less severe increase in the velocity dependence (over the U^8 law) than the self noise.

2. Sources in the presence of mean shear

Lighthill (1952) shows under certain restrictions that the density ρ at a position $\mathbf{x}$ far from a region of turbulence being convected at a velocity aM through a stationary medium is

$$\rho(\mathbf{x})-\rho_0 = \frac{x_i x_j}{4\pi a^4|\mathbf{x}|^3}(1-M\cos\theta)^{-3}\int\frac{\partial^2}{\partial\tau^2}T_{ij}(\boldsymbol{\eta},\tau)\,d\boldsymbol{\eta}. \tag{1}$$

The moving frame of reference η_i in which the integration is performed is related to the fixed frame y_i by the relationship

$$y_i = \eta_i + aM\delta_{1i}\tau,$$

τ being the retarded time $\quad t-\left|\dfrac{\mathbf{x}-\mathbf{y}}{a}\right|.$

The time derivative $\partial/\partial\tau$ in (1) is performed with $\boldsymbol{\eta}$ held constant.

For flows where there is a large mean velocity it is useful to separate T_{ij} into two parts, that explicitly dependent on mean velocity and that which is the result of the turbulent fluctuations alone. Following Lighthill (1954, section 5), the time derivative of T_{ij} holding $\mathbf{y}$ constant can be transformed by the equations of motion (with the viscous stresses neglected) to give

$$\frac{\partial}{\partial t}\rho u_i u_j(\mathbf{y},t) = p\frac{\partial u_j}{\partial y_i}+p\frac{\partial u_i}{\partial y_j}-\frac{\partial}{\partial y_k}(\rho u_i u_j u_k+\delta_{ki}pu_j+\delta_{kj}pu_i). \tag{2}$$

The relation connecting the two derivatives $\partial/\partial\tau$ and $\partial/\partial t$ has been discussed by Williams (1963) and we use his equation (1.27) to obtain

$$\int\frac{\partial}{\partial\tau}\bigg|_{\eta}\rho u_i u_j(\boldsymbol{\eta},\tau)\,d\boldsymbol{\eta} = (1-M\cos\theta)\int\frac{\partial}{\partial t}\bigg|_{y}\rho u_i u_j(\boldsymbol{\eta},\tau)\,d\boldsymbol{\eta}. \tag{3}$$

The third term in (2) is a space derivative and so, when substituted in the volume integral of (3), leads to an octupole source which is of the order M^2 smaller than the quadrupole terms retained. With the remaining two terms of (2) substituted, (1) becomes

$$\rho(\mathbf{x})-\rho_0 = \frac{x_i x_j}{4\pi a^4|\mathbf{x}|^3}(1-M\cos\theta)^{-2}\int \frac{\partial}{\partial\tau}\bigg|_\eta \left(p\frac{\partial u_j}{\partial y_i}+p\frac{\partial u_i}{\partial y_j}\right)(\boldsymbol{\eta},\tau)\,d\boldsymbol{\eta}. \tag{4}$$

This result differs from that of Lighthill by a factor $(1-M\cos\theta)$ as he did not correctly take account of the difference between the fixed- and moving-frame derivatives in (1) and (2).

If we separate the velocity into a fluctuating and a mean component relative to the undisturbed atmosphere, i.e. $u_i = U_i + u_i'$, the integrand in (4) can be separated into the term discussed by Lighthill

$$\frac{\partial}{\partial\tau}\left(p\frac{\partial U_i}{\partial y_j}+p\frac{\partial U_j}{\partial y_i}\right) \tag{5}$$

and a second term

$$\frac{\partial}{\partial\tau}\left(p\frac{\partial u_i'}{\partial y_j}+p\frac{\partial u_j'}{\partial y_i}\right).$$

There is, however, implicit dependence on mean velocity in this second term. An expression similar to (2) formed from the velocity fluctuations and the momentum equation reads

$$\frac{\partial}{\partial t}\bigg|_y \rho u_i' u_j' = p\frac{\partial u_j'}{\partial y_i}+p\frac{\partial u_i'}{\partial y_j}-\rho u_i' u_k\frac{\partial u_j}{\partial y_k}-\rho u_j' u_k\frac{\partial u_i}{\partial y_k}-\frac{\partial}{\partial y_k}(\delta_{ki}pu_j'+\delta_{kj}pu_i') - u_i'u_j'\frac{\partial \rho u_k}{\partial y_k}. \tag{6}$$

Within the volume integral we can neglect the divergences in (6) so that

$$p\frac{\partial u_i'}{\partial y_j}+p\frac{\partial u_j'}{\partial y_i}$$

can be replaced by

$$\frac{\partial}{\partial t}\bigg|_y \rho u_i' u_j' + \frac{\partial}{\partial y_k}\rho u_i' u_k' u_j' + \frac{\partial}{\partial y_k}\rho u_i' U_k u_j' + \rho u_i' u_k' \frac{\partial U_j}{\partial y_k} + \rho u_j' u_k' \frac{\partial U_i}{\partial y_k} + \rho u_i' U_k \frac{\partial U_j}{\partial y_k} + \rho u_j' U_k \frac{\partial U_i}{\partial y_k}. \tag{7}$$

The first term in (7), when substituted in (4), becomes

$$\frac{x_i x_j}{4\pi a^4|\mathbf{x}|^3}(1-M\cos\theta)^{-2}\int \frac{\partial}{\partial\tau}\bigg|_\eta \frac{\partial}{\partial t}\bigg|_y \rho u_i' u_j'(\boldsymbol{\eta},\tau)\,d\boldsymbol{\eta},$$

which with the aid of (3) can be written

$$N = \frac{x_i x_j}{4\pi a^4|\mathbf{x}|^3}(1-M\cos\theta)^{-3}\int \frac{\partial^2}{\partial\tau^2}\bigg|_\eta \rho u_i' u_j'(\boldsymbol{\eta},\tau)\,d\boldsymbol{\eta}. \tag{8}$$

This is the familiar 'self noise' term.

The second and third terms in (7) are divergences and so will be neglected. The remaining four terms when substituted in (4) become

$$\frac{2x_i x_j}{4\pi a^4|\mathbf{x}|^3}(1-M\cos\theta)^{-2}\int\frac{\partial U_i}{\partial y_k}\frac{\partial}{\partial\tau}\bigg|_\eta(\rho u_j' u_k' + \rho u_j' U_k)(\boldsymbol{\eta},\tau)\,d\boldsymbol{\eta}.$$

The term containing the momentum flux resulting from the product of fluctuating velocities was identified by Csanady (1966). The other term, the rate of change of momentum in the presence of a mean velocity gradient, i.e.

$$\frac{\partial U_i U_k}{\partial y_k}\frac{\partial}{\partial\tau}\rho u_j',\dagger$$

would be octupole and negligible to this order if $\partial U_i U_k/\partial y_k$ were constant (by the principle of conservation of momentum). To emphasize the non-zero part of the integral we can manipulate this term with the aid of (3) to give the fixed-frame time derivative

$$\frac{2x_i x_j}{4\pi a^4|\mathbf{x}|^3}(1-M\cos\theta)^{-1}\int\frac{\partial U_i U_k}{\partial y_k}\frac{\partial}{\partial t}\bigg|_y\rho u_j'(\boldsymbol{\eta},\tau)\,d\boldsymbol{\eta}. \tag{9}$$

An application of the momentum equation gives

$$-\frac{2x_i x_j}{4\pi a^4|\mathbf{x}|^3}(1-M\cos\theta)^{-1}\int\frac{\partial U_i U_k}{\partial y_k}\frac{\partial}{\partial y_l}(\rho u_j u_l + p\delta_{jl})(\boldsymbol{\eta},\tau)\,d\boldsymbol{\eta}.$$

If the integrand is grouped as a divergence, the only quadrupole part remaining is

$$\frac{2x_i x_j}{4\pi a^4|\mathbf{x}|^3}(1-M\cos\theta)^{-1}\int\frac{\partial^2 U_i U_k}{\partial y_l\partial y_k}(\rho u_j u_l + p\delta_{jl})(\boldsymbol{\eta},\tau)\,d\boldsymbol{\eta}. \tag{10}$$

Equation (10) tends to suffer from the same difficulty as (9), though less obviously, in that the integrand contains momentum terms of the form

$$\left[U_j\frac{\partial^2 U_i U_k}{\partial y_l\partial y_k}\right]\rho u_l',$$

which, for a constant value of the part in square brackets, might integrate to zero.

Grouping all the shear noise terms, one gets

$$S = \frac{2x_i x_j}{4\pi a^4|\mathbf{x}|^3}(1-M\cos\theta)^{-2}\int\frac{\partial U_i}{\partial y_k}\frac{\partial}{\partial\tau}(\rho u_j' u_k' + p\delta_{kj})(\boldsymbol{\eta},\tau)\,d\boldsymbol{\eta}$$
$$+\frac{2x_i x_j}{4\pi a^4|\mathbf{x}|^3}(1-M\cos\theta)^{-1}\int\frac{\partial^2 U_i U_k}{\partial y_l\partial y_k}(\rho u_j u_l + p\delta_{jl})(\boldsymbol{\eta},\tau)\,d\boldsymbol{\eta}. \tag{11}$$

Equations (8) and (11) show that the acoustic fluctuations can be considered the result of three different types of terms, self noise which depends on the second time derivative and shear noise which depends on two types of terms, one with the first derivative of velocity and one involving the second derivative of velocity. The self noise term becomes increasingly important at higher convection Mach numbers because of the factor $(1-M\cos\theta)^{-3}$.

† Incompressible flow for the velocities U_i is assumed.

3. Sources in an idealized jet

The expression for the acoustic intensity becomes very lengthy since it is formed from the products of all the above terms. In order to have some idea of the principal terms in the practical case of a round jet, an idealized flow is postulated. Near the centre-line of a self-preserving mixing layer, springing from a nozzle (see figure 1 for co-ordinates), the flow is two-dimensional with only one mean gradient $\partial U_1/\partial y_2$ and no lateral velocity U_2. As the mean velocity has a point of inflexion near the mixing layer centre-line, the velocity gradient is assumed to be constant over the region of non-zero correlation. Away from the mixing layer centre-line this idealization becomes increasingly unrealistic, but this may be of little consequence because the most efficient noise generators are located near the centre of the mixing layer.

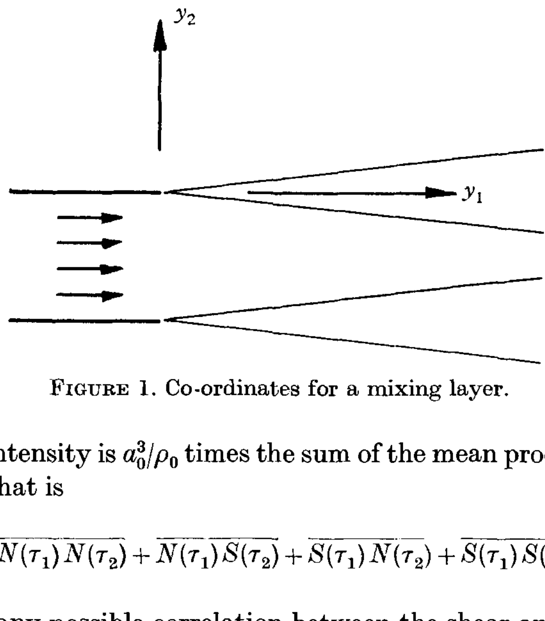

FIGURE 1. Co-ordinates for a mixing layer.

The acoustic intensity is a_0^3/ρ_0 times the sum of the mean products of the terms in (8) and (11), that is

$$\frac{a^3}{\rho_0}\{\overline{N(\tau_1)N(\tau_2)}+\overline{N(\tau_1)S(\tau_2)}+\overline{S(\tau_1)N(\tau_2)}+\overline{S(\tau_1)S(\tau_2)}\}. \tag{12}$$

We shall ignore any possible correlation between the shear and self-noise terms and, as we are considering a flow where the second derivative of the mean velocity is zero, the terms in the second part of (11) are zero. For such a flow the intensity becomes

$$\begin{aligned} I(\mathbf{x}) = {} & \frac{x_i x_j x_l x_m}{16\pi^2 a^5 \rho|\mathbf{x}|^6(1-M\cos\theta)^5}\iint \frac{\partial^4}{\partial\tau^4}\overline{\rho u_i' u_j'(\boldsymbol{\eta},0)\,\rho u_l' u_m'(\boldsymbol{\eta}+\boldsymbol{\zeta},\tau)}\,d\boldsymbol{\eta}\,d\mathbf{y} \\ & + \frac{x_1 x_j x_1 x_m}{4\pi^2 a^5\rho|\mathbf{x}|^6(1-M\cos\theta)^3} \\ & \times \iint \left(\frac{\partial U_1}{\partial y_2}\right)^2 \frac{\partial^2}{\partial \tau^2}\overline{(p\delta_{2j}+\rho u_j' u_2')(\boldsymbol{\eta},0)\,(p\delta_{2m}+\rho u_m' u_2')(\boldsymbol{\eta}+\boldsymbol{\zeta},\tau)}\,d\boldsymbol{\eta}\,d\mathbf{y}, \end{aligned} \tag{13}$$

where, since we wish to consider the radiation from a finite volume of turbulence, the outer integration is with respect to stationary co-ordinates (for further discussion see Williams 1963).

Equation (13) contains the two convection factors $(1-M\cos\theta)^{-5}$ and $(1-M\cos\theta)^{-3}$ that are applicable for the idealized mixing layer. If second derivatives of the mean velocity occur there is the additional factor $(1-M\cos\theta)^{-1}$.

4. Total power output from convected sources

Convection of the sound sources increases the total power radiated, the increase in power over a stationary source being given by the surface integral

$$\int \frac{x_i x_j x_l x_m}{|\mathbf{x}|^6(1-M\cos\theta)^n}\, dS,$$

where $n = 5$ for self noise and $n = 3$ or 1 for shear noise.

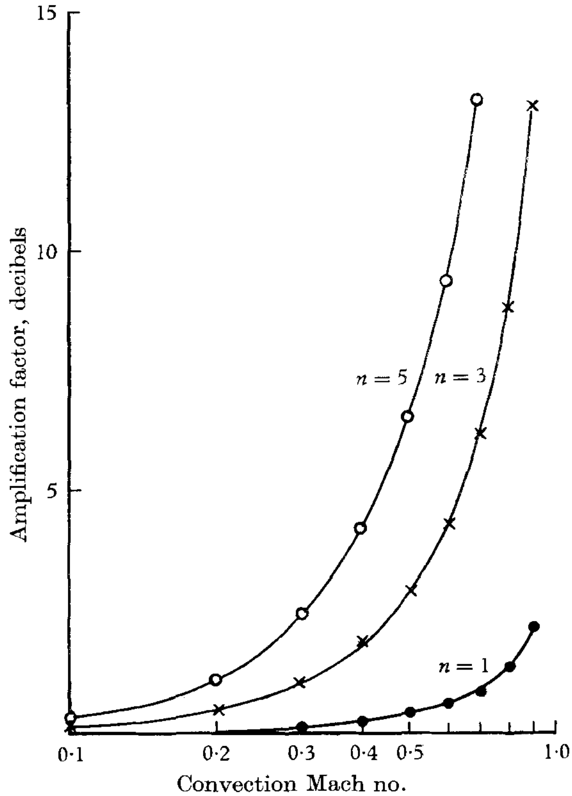

FIGURE 2. Amplification of total radiation due to convection term $(1-M\cos\theta)^{-n}$, for a lateral quadrupole.

This integral has been evaluated for a lateral quadrupole with $i = j = 1$, $l = m = 2$ for both self noise and shear noise. The amplification by convection of the total radiation compared with that from this source at $M = 0$ is shown in figure 2. The substantial reduction in amplification for shear noise is evident.

5. Experimental directionality patterns

Howes (1960) has collected the results of a number of experiments to determine the directionality patterns of small air jets. We can compare these results with the two convection patterns of (13), $(1-M\cos\theta)^{-3}$ and $(1-M\cos\theta)^{-5}$. Wills (1964) has measured the convection velocity at the centre of a mixing layer (where one would expect most of the sound to originate) and found it to vary slowly, being about 0·63 of the exit velocity. Thus, for the range of exit velocities Howes used, the convection Mach number lies between $M = 0{\cdot}5$ to $M = 0{\cdot}62$.

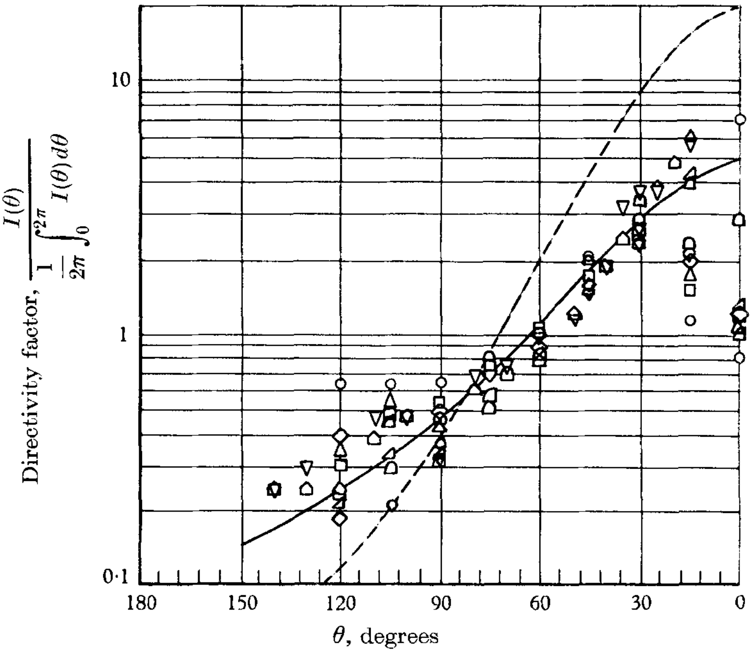

FIGURE 3. Directionality patterns fitted to Howes's (1960) data. Symbols as in his figure 16. ——, $(1-0{\cdot}55\cos\theta)^{-3}$; – – – –, $(1-0{\cdot}55\cos\theta)^{-5}$.

Figure 3 shows an attempt to fit the two convection patterns $(1-M\cos\theta)^{-3}$ and $(1-M\cos\theta)^{-5}$ (multiplied by the same arbitrary constant) to Howes' results. We have, of course, neglected such phenomena as the refraction of the sound and the preferred orientation of the quadrupoles but the result is still of importance as it shows that the new shear noise factor $(1-M\cos\theta)^{-3}$ provides a much better fit. The large reduction in intensity at small angles of θ is probably due to refraction.

6. Conclusions

The quadrupole forcing function representing the generators of aerodynamic noise is dependent upon several different mechanisms, fluctuating stresses acting on themselves, fluctuating stresses in the presence of a mean shear and fluctuating stresses in the presence of the second derivative of mean velocity. In a round jet, only the first two of these are apparently of any practical significance, the second one possibly being dominant. Each of these mechanisms has a different convection factor associated with it. The factor $(1-M\cos\theta)^{-3}$ for the shear noise term involving the first mean velocity derivative shows a reasonable agreement with the experimental results on round jets.

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